

## REPORT DOCUMENTATION PAGE

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13. ABSTRACT (Maximum 200 words)  Three design tools for hybrid optical/digital imaging systems have been developed that provide new ways of designing the non-conventional imaging systems that are optimum for extended depth of field/focus, and for passively determining the range to objects in a scene. The first tool developed was a defocus transfer function (DTF) for imaging systems with radially symmetric pupils. The DTF allows one to visualize how the modulation transfer function (MTF) will change with focus. Prior work was valid only for one-dimensional optical systems. The second tool uses a form of a Radon-Wigner distribution to show the impulse response, or point spread function, as the focus changes. The third tool is the use of linear transformation operators to analyze and simulate optical systems. These three tools are proving to be very powerful in the design of hybrid optical/signal processing systems.			
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**(Continuation Sheet)**

**1. LIST OF MANUSCRIPTS**

Alan R. FitzGerrell and W. Thomas Cathey, "A Defocus Transfer Function for Imaging Systems with Radially Symmetric Pupils," accepted for publication by Applied Optics.

**2. SCIENTIFIC PERSONNEL**

Alan FitzGerrell, W. Thomas Cathey, Sara Bradburn

**HONORS / AWARDS / DEGREES**

Alan FitzGerrell received the degree Master of Science in Electrical Engineering in 1996.

W. Thomas Cathey was made a Fellow of the IEEE in 1996.

Sara Bradburn received the 1996 Colorado Technology Transfer Award from the Colorado Advanced Technology Institute.

W. Thomas Cathey, Edward R. Dowski, and Alan R. FitzGerrell received the 1996 University-to-Industry Technology Transfer Award from the Colorado chapter of the Technology Transfer Society.

**3. INVENTIONS**

None

**4. SCIENTIFIC PROGRESS AND ACCOMPLISHMENTS**

Work on the focal invariant system is continuing, with emphasis on the invariance of such imaging systems to focus related aberrations. These include curvature of field and chromatic aberration, which are important aberrations in wide-field-of-view systems. An attached paper summarizes those results. Three design tools are in various stages of development. The defocus transfer function (DTF) was described in an earlier report, in which it was called the quasi-ambiguity function, because of its similarity to the Woodward ambiguity function of radar. The DTF displays the optical transfer function as a function of the degree of misfocus of an imaging system, and is a tool that is useful for analysis and design of extended depth of focus or passive ranging systems that have circular symmetry. A paper describing this was attached to a previous report, and has been accepted by Applied Optics. More recent work is on a means of designing in the point spread function (PSF) domain. The Radon-Wigner distribution is one that can be used to observe the PSF as a function of misfocus. The Wigner distribution and the Ambiguity function are related via an autocorrelation function, where the Fourier transform of the autocorrelation function is the Wigner Distribution, and the inverse Fourier transform is the ambiguity function. This work has just begun, but promises to provide design tools so that the optical system can be designed to give a desired PSF throughout a range of misfocus. A short paper describing this work is attached. Another design tool simulates optical systems with linear transformations and matrix representations. This is proving to be quite useful in the design of passive ranging systems, or other optical systems where the detector is a discrete detector array, such as a CCD array.

**5. TECHNOLOGY TRANSFER**

Cooperative research has been carried out with Joe van der Gracht, ARL, Alelphi, MD. The research group at the University of Colorado received a total of three awards for transfer of technology from the university to industry.

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# Aberration-Invariant Optical/Digital Imaging Systems

W. Thomas Cathey, Edward R. Dowski, Jr.

Imaging Systems Laboratory

Department of Electrical & Computer Engineering

University of Colorado

Boulder, Colorado 80309-0425

<http://www-ocs.colorado.edu/isl>

There has existed a classic and fundamental trade-off in imaging systems between light gathering ability (or aperture size) and depth of field (or depth of focus). Implicit in this trade-off is the notion that the imaging system forms the *final* image; no processing of the image is performed. This trade-off is so well known that it is easy to overlook the fact that it might not hold for imaging systems that include processing to form the final image. In fact, the Imaging Systems Laboratory has both theoretically proven and experimentally demonstrated that the traditional trade-off between aperture size and depth of field is not valid for imaging systems modified by a special-purpose wavefront mask or phase plate and the resulting imagery processed to produce the final image [1,2,3]. We have developed a new type of optical/digital imaging system that can produce the large depth of field normally associated with a small aperture system and the light gathering power of a large aperture system.

A large aperture/large depth of field system is a specific example of what we call aberration-invariant systems. Aberration-invariant systems are general and can be applied to increase the depth of field of an imaging system, increase the field of view of a system, or control focus-related aberrations of lens systems, such as curvature of field and chromatic aberration [4,5]. Physically, aberration-invariant imaging systems are traditional imaging systems modified with a special-purpose optical mask or phase plate. The resulting image, or intermediate image, is then processed to produce the final image.

By making an imaging system invariant to aberrations, it is possible to make imaging systems that are cheaper and have properties

that are otherwise not obtainable. For example, with tolerance to misfocus, video cameras, digital cameras, and orthoscopes need not be focused. With tolerance to field curvature, the field of view of a flat bed scanner can be increased and with a simpler optical system. With tolerance to chromatic aberration an imaging system, especially ones with plastic lenses, can be made achromatic with much simpler optics.

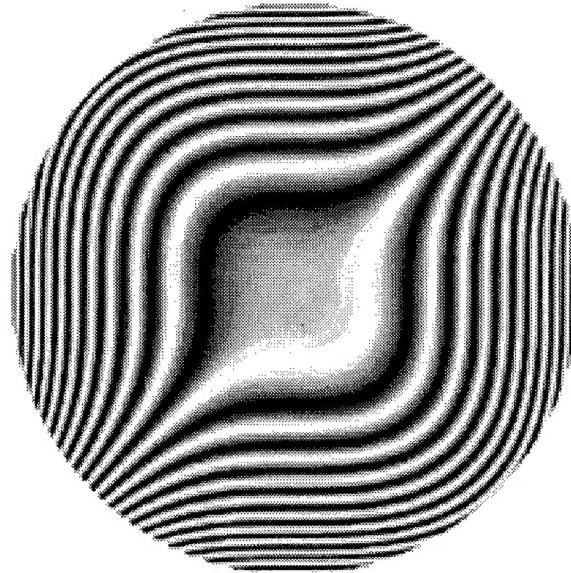


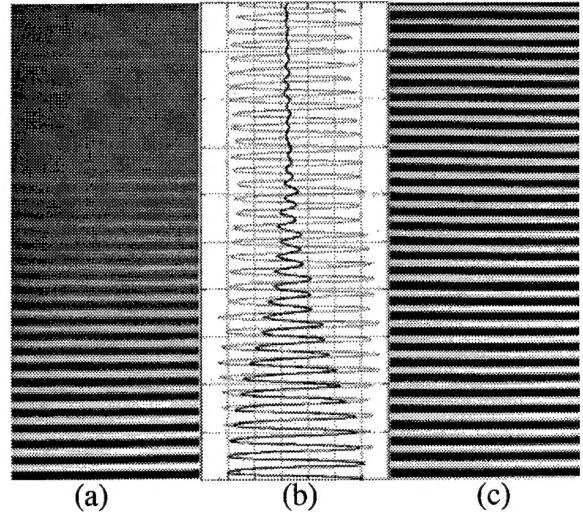
Figure 1: Interferogram of aberration-invariant wavefront mask. This mask has a cubic-phase structure with a phase deviation of about 20 wavelengths on the horizontal and vertical axes.

One way to understand the operation of aberration-invariant imaging is through analogy to communication systems. Many types of information are coded for transmission or storage and later decoded when received or retrieved. This coding/decoding process allows the final

decoded information to be insensitive to errors that may occur between the coder and decoder. Wireless communication systems, for example, tailor the systems' impulse response to code the transmitted signal in such a way that after decoding, interference or multipath errors are rarely noticed. Information stored on compact disks is also coded before being written. Coding the information makes the system insensitive to bit errors on the disk medium. In a similar way, coding an imaging wavefront with a special-purpose wavefront mask can modify the point spread function such that the formed image is insensitive to a number of errors or aberrations commonly found in imaging systems [6,7]. The resulting or intermediate image is not directly a sharp, clear image. But, the image remains largely unaltered with changes in the imaging system aberrations. Processing of the image decodes the coded image and produces a final sharp, clear image.

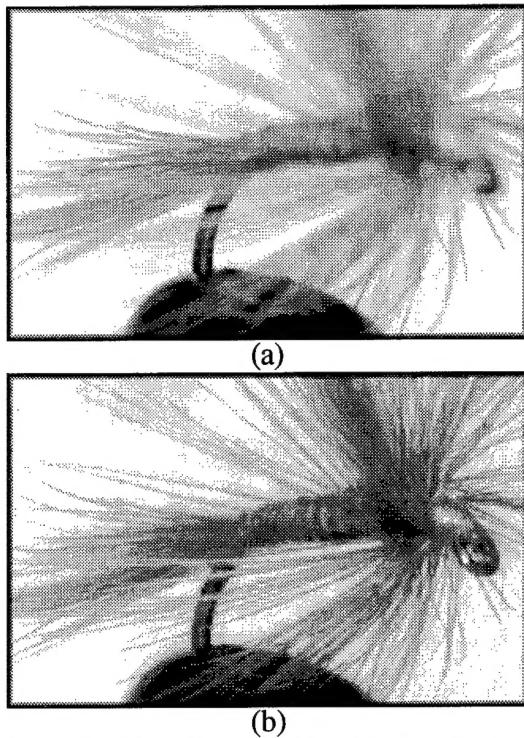
The special-purpose wavefront mask is not rotationally symmetric and has no optical power in the traditional imaging sense. But, the addition of this mask to a traditional imaging system controls the imaging system such that dramatic changes in a large number of parameters of the system have little or no effect on the final image. There are a large number of wavefront masks that can have this property. One mask that has this property and also has the additional property of allowing efficient processing or decoding of the intermediate image is described by a rectangularly-separable cubic-phase structure. A traditional method of visualizing optical surfaces is through interferograms. An interferogram of the cubic-phase mask is given in figure 1. The number of fringes in the interferogram, and hence the number of wavelengths of optical path difference can vary from twenty to over one hundred depending on the application.

Examples of aberration-invariant imaging are easiest with images that are focus-invariant. A quantitative example of aberration-invariant imaging for increased depth of field can be seen when imaging an inclined bar chart. See figure 2.



*Figure 2: Imaging inclined bar charts. Imaging a bar chart inclined at sixty degrees from vertical with a traditional imaging system produces the image of (a). Modifying the imaging system with a cubic-phase mask and processing the resulting image results in a final image shown in (c). Traces of the center of images (a) and (c) are shown in (b). Notice the phase reversal that the image from the traditional system undergoes with large misfocus.*

With best focus of the bar chart being at the bottom of the chart, the upper bars undergo increasing degrees of misfocus. By the top of the chart imaging with the traditional system, seen in figure 2(a), is severely out of focus. With the inclusion of the cubic-phase mask and processing of the resulting intermediate image, the corresponding image of figure 2(c) results. This image is sharp and clear over the entire space of the inclined bar chart. Figure 2(b) shows two traces from the center columns of the traditional and aberration-invariant images. The darker trace is from the traditional system and clearly suffers from misfocus. The trace from the aberration-invariant system has almost no change in performance over the entire bar chart. At very large values of misfocus traditional systems undergo phase reversals; white bars image as black bars and vice versa. This phase reversal can be seen about one third down from the top of the image.



*Figure 3: Imaging artificial fly-fishing flies. Macro imaging of small objects traditionally requires a large depth of field. A traditional  $f = 80 \text{ mm}$   $F/7$  imaging system when focused close has a very small depth of field (a). Only a small region near the tail of the fly is in focus. By modifying the system with a special-purpose optical phase mask and post-processing the resulting imagery, the image of (b) is produced. This image shows a much larger depth of field than that from the traditional system.*

The effects of general greyscale imaging with aberration-invariant systems can be quickly noticed when applied to macro-scale imaging. In macro imaging the focus position is very near the lens and the depth of field of the system is near its minimum. Figure 3 is an example of macro imaging with a traditional imaging system and a focus-invariant imaging system. Figure 3(a) is a image of a fly-fishing fly. The lens used is a two-lens  $f = 80 \text{ mm}$ ,  $F/7$  with the fly about 200 mm from the first lens. In this configuration the depth of field of the resulting image is much less than the width of the fly, as seen in figure 3(a). By modifying the imaging system with a special-purpose phase cubic-phase mask, and digitally filtering the resulting imagery, the image of

figure 3(b) results. This image is in focus over the entire width of the fly.

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## OTF AND PSF RELATIONSHIPS IN BILINEAR DOMAINS

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Gregory E. Johnson  
Imaging Systems Laboratory  
Department of Electrical and Computer Engineering  
University of Colorado at Boulder  
Boulder, Colorado 80309-0525

It has been shown that the frequency response of an optical system (the Optical Transfer Function, or OTF) can be represented for all values of misfocus through the Ambiguity Function description of the system's aperture function [1]. We can also show that the *spatial*-domain response of the system (the Point Spread Function, or PSF) can be represented for all values of misfocus through the Radon-Wigner Distribution (RWD). Analytical procedures applicable to both domains would provide powerful design tools using either representation.

### 1. BILINEAR DESCRIPTIONS

The Wigner Distribution (WD) and Ambiguity Function (AF) provide bilinear phase-space representations of signals. In this document, one-dimensional formulations are presented. The WD and AF of a signal  $s(x)$  are based on the *instantaneous autocorrelation* function  $R(x, x')$  [2], shown in Equation 1.1.

$$R(x, x') = s(x + x' / 2)s^*(x - x' / 2) \quad (1.1)$$

The WD  $W(x, v)$  is represented as the Fourier transform of  $R(x, x')$  with respect to the lag variable  $x'$ , while the AF  $A(x', u)$  is generated by the Inverse-Fourier transform of  $R(x, x')$  with respect to the space variable  $x$ . These are shown in Equations 1.2 and 1.3.

$$W(x, v) = \int R(x, x') e^{-j2\pi vx'} dx' \Leftrightarrow R(x, x') = \int W(x, v) e^{j2\pi vx'} dv \quad (1.2)$$

$$A(x', u) = \int R(x, x') e^{j2\pi ux} dx \Leftrightarrow R(x, x') = \int A(x', u) e^{-j2\pi ux} du \quad (1.3)$$

### 2. FREQUENCY DOMAIN: AF

For an in-focus incoherent optical system, the OTF  $H(f)$  is represented as the integration of the instantaneous autocorrelation of the pupil function [1], shown in Equation 2.1. Here the frequency variable  $f$  has been inserted in place of the spatial coordinate  $x'$ .

$$H(f) = \int R(x, x') dx \Big|_{f=1/x'} \quad (2.1)$$

For a *defocused* system, the OTF  $H(f)$  is represented as shown in Equation 2.2, where the phase-factor from misfocus has been included.

$$H(f; \psi) = \left[ \int R(x, x') e^{j2\pi x(x'\psi)} dx \right] \Big|_{f=1/x'} \quad (2.2)$$

where  $\psi = \frac{2}{\lambda} W_{20}$  is the degree of misfocus

Equation 2.2 is seen as being similar to the AF description in Equation 1.3. The exact relationship is shown in Equation 2.3.

$$H(f; \psi) \rightarrow A(x', u) \Big|_{\substack{u=x'\psi \\ f=1/x'}} = A(x', x'\psi) \Big|_{f=1/x'} \quad (2.3)$$

Here  $H(f)$  is interpreted as a slice out of the AF at an angle related to  $\psi$ , projected to the horizontal axis. This is shown graphically in Figure 2.1 for a rectangular aperture.

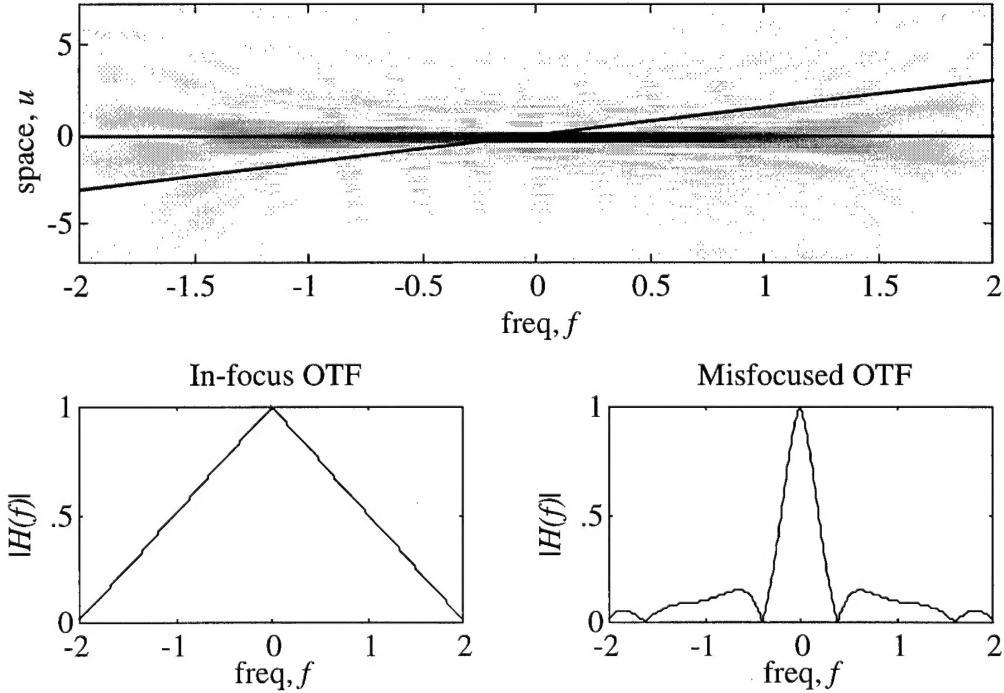


Figure 2.1. OTF  $H(f)$  as a slice of the Ambiguity Function for a rectangular aperture.

### 3. RADON TRANSFORM: INTRO

Before proceeding to the PSF and spatial domain representations, the Radon transform requires introduction. The Radon transform allows an object  $b(x,y)$  to be represented by integrated slices

through the  $x,y$  coordinate system through a distribution of angles. This transform is shown analytically in Equation 3.1 and graphically Figure 3.1.

$$\begin{aligned} g(s, \theta) &= \int b(x, y) \Big|_{\substack{x=s \cos \theta - u \sin \theta \\ y=s \sin \theta - u \cos \theta}} du \\ &= \int b(s \cos \theta - u \sin \theta, s \sin \theta - u \cos \theta) du \end{aligned} \quad (3.1)$$

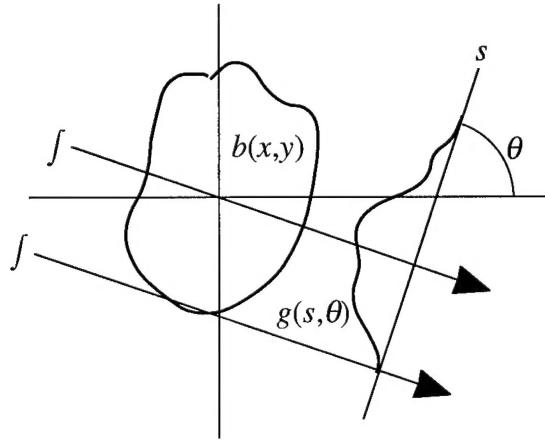


Figure 3.1. Radon Transform  $g(s, \theta)$  of an object  $b(x, y)$

At each value of  $\theta$ , the object  $b$  is integrated to produce a projection  $g$ . Note that this transform is linear and has a variety of properties regarding multiplication, convolution, and FT relationships [3].

#### 4. SPATIAL DOMAIN: RWD

The OTF description of the system via the Ambiguity Function can be extended to a PSF description by considering the Inverse-Fourier transform (IFT) of the OTF to change domains. The IFT of  $H(f)$  to generate  $h(z)$  is shown in Equation 4.1, where the AF representation of  $H(f)$  has been inserted.

$$h(z; \psi) = \int H(f; \psi) e^{+j2\pi f} df = \iint R(x, x') e^{+j2\pi x'(x\psi + z)} dx' dx \quad (4.1)$$

We can rewrite the Wigner Distribution function from Equation 1.2 as shown in Equation 4.2, which leads to a Radon-Wigner Distribution (RWD) interpretation of the PSF in Equation 4.1.

$$W(x, v) = \int R(x, x') e^{+j2\pi x'(-v)} dx' \quad (4.2)$$

From the two relationships in Equations 4.1 and 4.2 it is seen that the PSF  $h(z)$  for a misfocus  $\psi$  can be interpreted as a scaled Radon transform of the Wigner distribution of the aperture function, indicated in Equation 4.3 and graphically in Figure 4.1.

$$h(z; \psi) \rightarrow \int W(x, v)|_{v=-(x\psi+z)} dx = \int W(x, -x\psi - z) dx \quad (4.3)$$

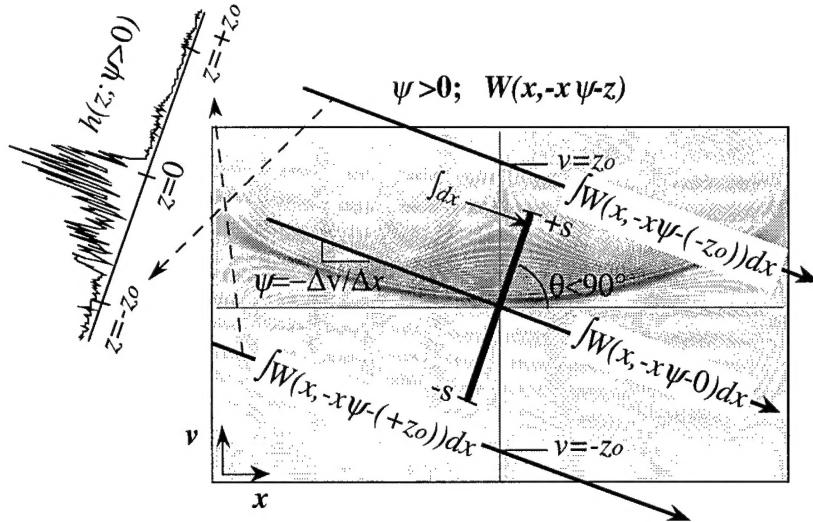


Figure 4.1. A Radon-Wigner transform for a Cubic-Phase Mask aperture function

In other words, the PSF or spatial characteristics of the optical system can now be fully described through all degrees of misfocus by Radon-transforming and scaling the Wigner Distribution. This special distribution is cited frequently in literature as the Radon-Wigner Distribution, or RWD, and has a variety of useful properties for both signal analysis and signal synthesis [3].

## 5. CONCLUSIONS

There remains the issue of scaling to formalize these relationships. While the AF provides a frequency domain description, the interpretation of OTF requires projection of the energy values to the horizontal axis of the AF. This is in effect a scaling on the  $f$ -axis that is related to the cosine of the angle formed by the misfocus parameter. In the RWD representation of the PSF, a true Radon transform must also have the  $s$ -axis scaled by an amount related to the misfocus parameter in order to properly interpret the PSF. Also, in the RWD there is a coordinate inversion on the  $s$ -axis. These RWD scalings and the AF scaling should be related to each other through the Fourier Transform that relates  $H(u)$  and  $h(x)$ .

While these analyses provide powerful analytical relationships, generating useful tools for system design requires signal synthesis methodologies. The WD and AF are both invertable to produce a unique signal, to within a constant phase factor. Unique signal reconstruction from the Radon representation is also possible. Considerable work has been performed to provide more general synthesis tools using the AF and WD, as well as the RWD [4]-[7]. However, the ability to perform optical system design using these domains remains undeveloped.

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